

*Title:*  **$^{235}\text{U}(94\%)$  SPHERES SURROUNDED BY  
NATURAL-URANIUM REFLECTORS**

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Volume II

HEU-MET-FAST-032

**$^{235}\text{U}(94\%)$  SPHERES SURROUNDED BY  
NATURAL-URANIUM REFLECTORS**

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## **$^{235}\text{U}(94\%)$ SPHERES SURROUNDED BY NATURAL URANIUM REFLECTORS**

**IDENTIFICATION NUMBER:** HEU-MET-FAST-032

**KEY WORDS:** acceptable, critical experiments, highly enriched uranium, metal, natural uranium reflector, natural-uranium-reflected, sphere

### **1.0 DETAILED DESCRIPTION**

#### **1.1 Overview of Experiments**

In the early 1950s a series of experiments was performed at the Los Alamos critical assembly facility to determine the critical mass of highly enriched uranium spheres surrounded by thin natural uranium reflectors. The objective of these experiments was to obtain a precision graph of the critical mass of highly enriched uranium metal spheres as a function of natural-uranium reflector thickness and to generate transport cross sections for the reflector material. Four configurations were measured with the thickness of the natural-uranium reflectors ranging from approximately 0.7 in. to 3.9 in. All four configurations were subcritical with measured maximum multiplications ranging from 53 to 167.

The experiments are documented in References 1 - 4.

Based on this evaluation all four configurations were judged to be acceptable for use as criticality-safety benchmark experiments.

#### **1.2 Description of Experimental Configuration**

All the experiments were performed on the Comet assembly machine at the Los Alamos critical assembly facility. A photograph of a typical experiment on the Comet machine is shown in Figure 1, and a schematic of a test setup is shown in Figure 2. The upper reflector hemisphere was screwed to a hydraulic piston attached to an "A" frame secured to the Comet vertical support structure. The lower reflector hemisphere containing the enriched-uranium sphere was seated on a hollow aluminum cylinder attached to the top of an hydraulic ram. During assembly the upper reflector hemisphere remained stationary while the lower reflector and core were raised. Note that this experiment did not contain any midplane diaphragm often used in similar experiments.

The counting system consisted of three boron-lined neutron chambers placed in long-counter geometry and mounted on a lift situated several feet from the reflector surface. Any one of these monitors would initiate separation of the materials by dropping the ram and raising the hydraulic piston if the measured neutron leakage exceeded a preset level.

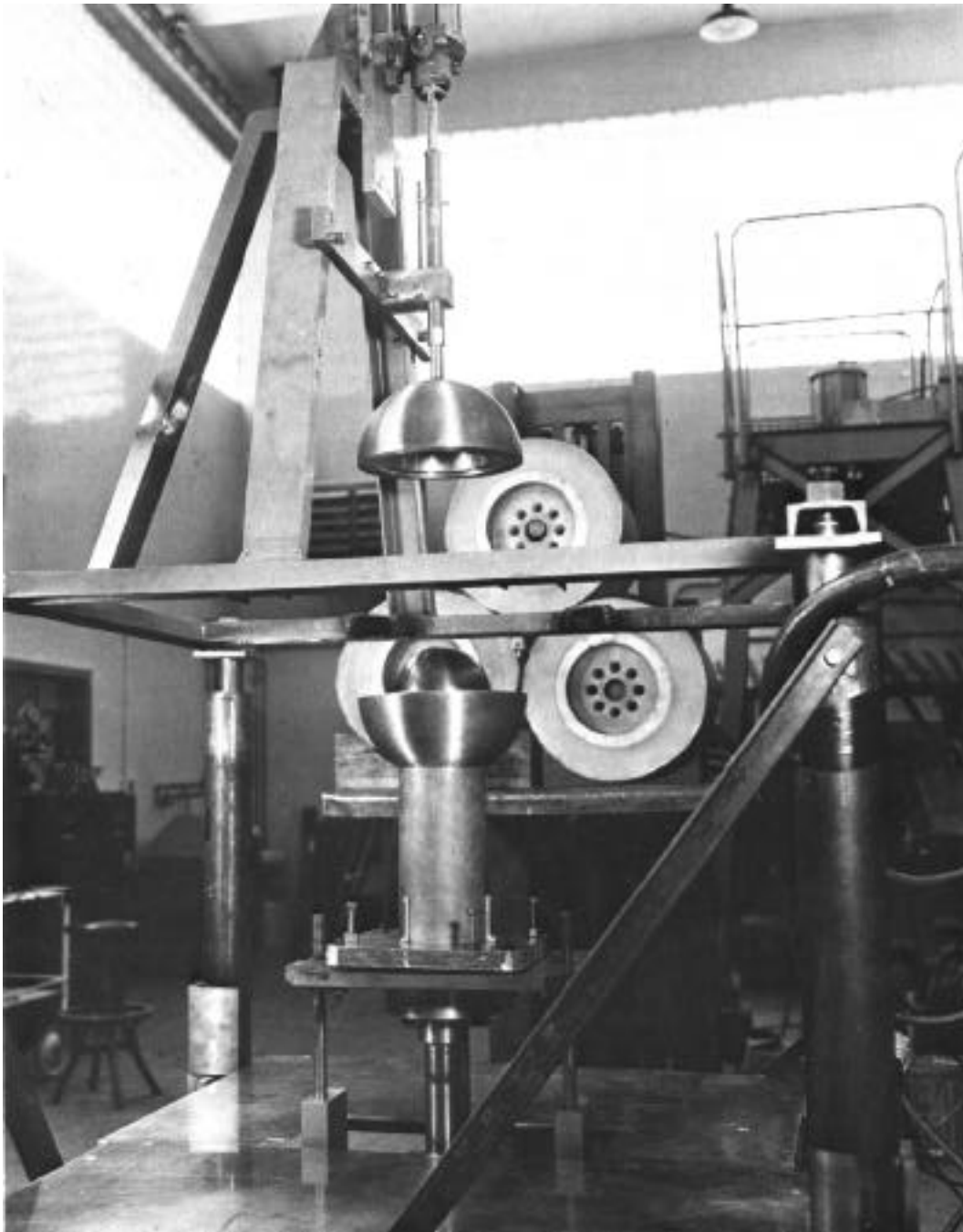


Figure 1. Photo of Comet Machine Showing Upper Reflector Hemisphere Suspended from "A" Frame and HEU Core and Lower Reflector Hemisphere Seated on Aluminum Cylinder.

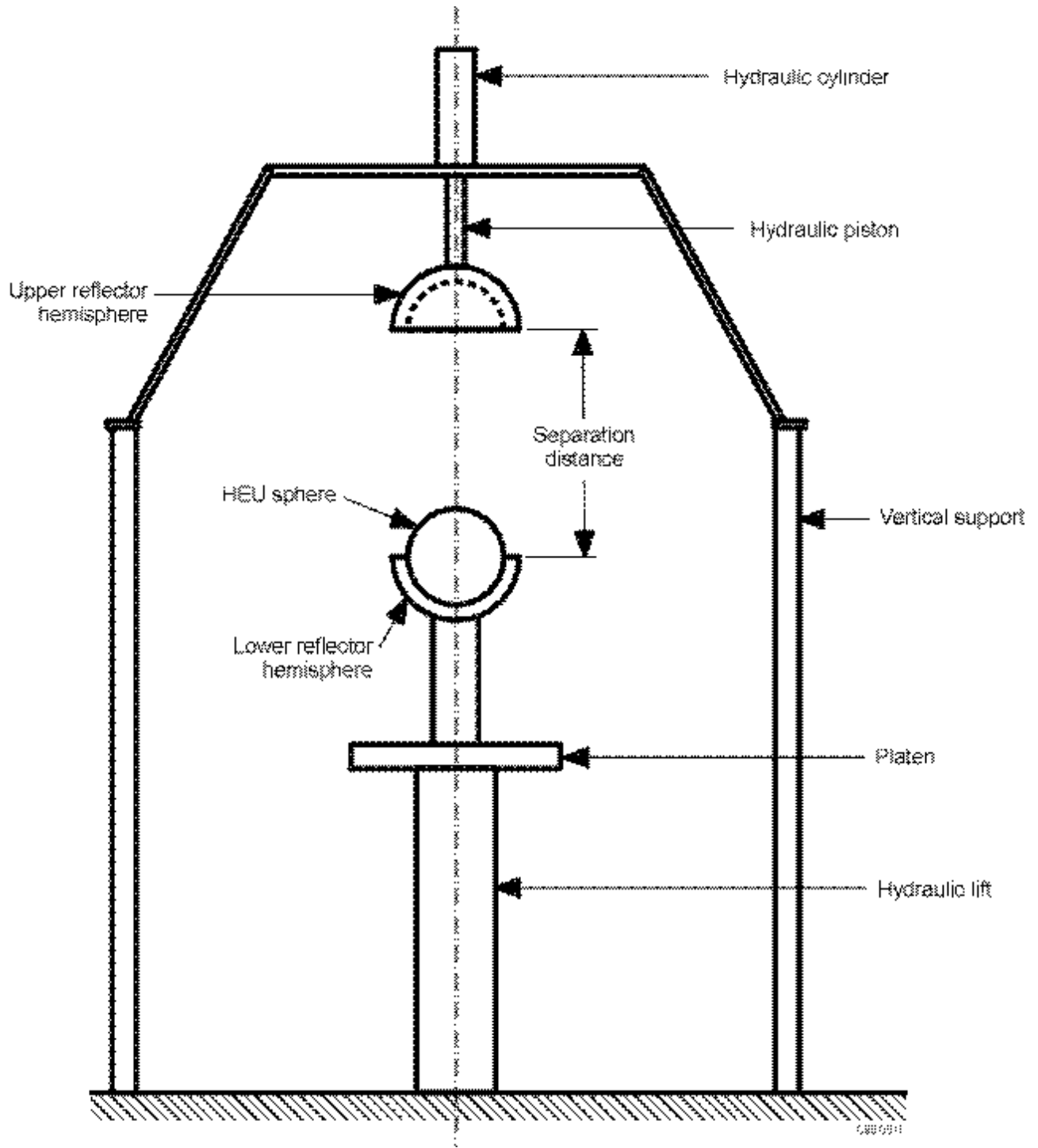


Figure 2. Schematic of HEU Sphere and Natural Uranium Reflector Experiment on the Comet Assembly Machine.

**Experimental Procedure** - A mock fission source was contained in a small central cavity (0.83-in. OD) of the core sphere. Two source-containing HEU filler pieces were used: a 70.4-g HEU sphere that contained a cylindrical source, and two HEU hemispheres with a total HEU mass of 84.0 g, that contained an approximately 0.34-in. central source cavity.

The HEU pieces for these measurements were in the form of nesting hemispherical shells. All of the HEU shells were duplicated with identical pieces (size and shape) of natural uranium. With the central source in place, an unmultiplied count was taken using the natural-uranium components. Next, the natural-uranium core pieces were replaced with the active HEU pieces and a multiplied count was obtained. The ratio of the multiplied count (with the HEU core) to the unmultiplied count (with the natural-uranium core) determined the assembly multiplication (M) for the measured configuration.

The lower hemispherical reflector and HEU sphere located on the hydraulic ram were raised in preset steps toward the stationary top reflector. Aluminum shims of various thicknesses were placed on the lower hemisphere to provide specific separation distances. Multiplication measurements were made as the separation of the hemispheres was progressively decreased, and the inverse multiplication (1/M) was plotted as a function of the separation distance, defined as the distance between the horizontal surfaces of the reflector hemispheres. The final measured configuration for three of the experiments (Cases 1, 2 and 3 in Table 1) was fully closed. A 0.026-in gap existed for the most reactive configuration of Case 4.

Based on measured multiplication changes associated with additions of HEU filler pieces to the central source cavity, corrections were applied to extrapolate the measured multiplication to a filled cavity.

An extrapolated critical mass was determined for each of the experiments (Reference 1). For Case 3 the critical mass was determined by a linear extrapolation of (1/M) versus HEU radius. For the other 3 cases the critical radii, and subsequently the corresponding critical masses, were determined from the Serber relation:

$$S = (1/M) / (r/r_0)$$

Where  $r$  is the observed core radius,  $r_0$  is the critical radius, and (1/M) is the reciprocal multiplication. The constant  $S$  was assumed to be approximately 1.15 based on previous measurements of bare, and natural-uranium-reflected HEU metal experiments.<sup>a</sup>

In addition to correcting for a filled central cavity and extrapolating to the critical mass, the experimenters also corrected for the effect of filling gaps within the core and between the core and reflector (Reference 3). Thus, the reported critical masses are for an idealized configuration of a spherical homogeneous core and reflector.

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<sup>a</sup> The Serber relationship for reflected systems were derived from measurements described in the following two reports: V. Josephson, "Critical Mass Measurements on Oy in Tu and WC Tampers," LA-1114 (May 1951), and V. Josephson, et. al., "Oralloy Shape Factor Measurements," LA-1155 (August 1950).

The critical specifications which were listed in References 1 - 4 are shown in Table 1.

Table 1. Critical Specifications for HEU/Natural-Uranium-Reflected Spheres.

Case Number	Ref. Number	Reflector Thickness (in)	Maximum Measured Multiplication	Extrapolated Critical Mass (kg U)	Extrapolated Critical Mass (kg <sup>235</sup> U)
1	1	3.925	0.0060 <sup>(a)</sup>	19.74	na
	2	3.925	167	19.83±0.5%	na
	3	3.93	na	19.82±0.5%	na
	4	3.93	167	na	18.61 ± 0.09
2	1	3.525	0.0188 <sup>(a)</sup>	20.47	na
	2	3.525	53	20.6±1%	na
	3	na	na	na	na
	4	3.52	53	na	19.2 ± 0.2
3	1	1.761	0.0071 <sup>(a)</sup>	26.45	na
	2	1.76	141	26.6±0.5%	na
	3	1.742	na	26.56±0.5%	na
	4	1.742	141	na	24.96±0.12
4	1	0.695	0.0064 <sup>(a)</sup>	36.41	na
	2	0.695	156	36.3±0.5%	na
	3	0.683	na	36.53±0.5%	na
	4	0.683	156	na	34.31 ± 0.17

(a) reciprocal multiplication (1/M)

na=not available

The four experimental configurations are shown in Figure 3.

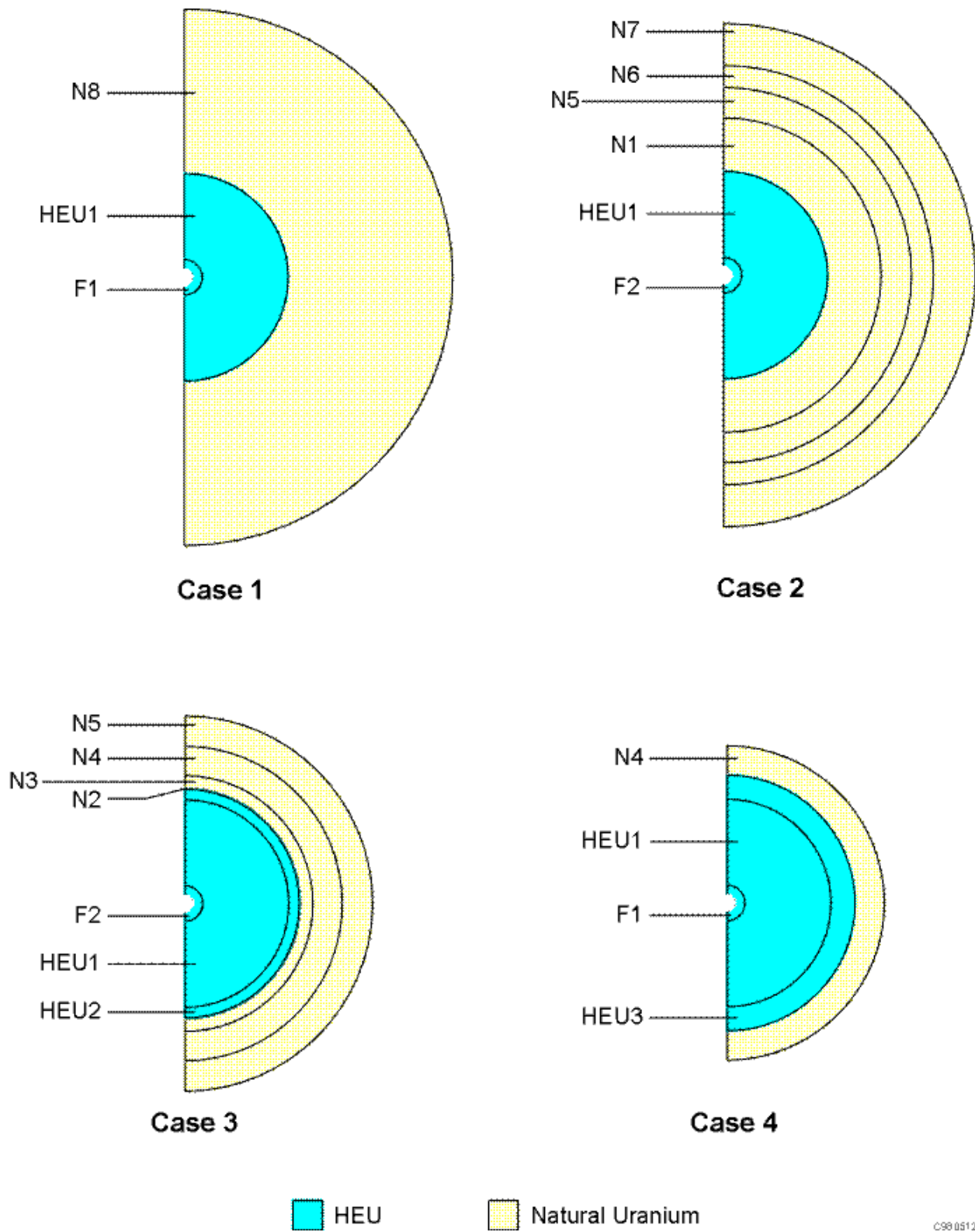


Figure 3. Experimental Configurations.



### 1.3 Description of Material Data

There are slight variations in the material data presented in the references. The basic material data are summarized in Table 2.

Table 2. Summary of Material Data for Critical Conditions of HEU/Natural-Uranium-Reflected Spheres.

Case Number	Reference Number	Core		Reflector
		HEU Density (g/cm <sup>3</sup> )	<sup>235</sup> U Fraction (wt.%)	Uranium Density (g/cm <sup>3</sup> )
1	1	18.76	93.90	na
	2	18.8	93.5	~19.0
	3	18.69	93.90	19.00
	4	18.69	93.9	19.00
2	1	18.76	93.90	na
	2	18.8	93.5	~19.0
	3	na	na	na
	4	18.75	93.9	19.0
3	1	18.74	93.99	na
	2	18.8	93.5	~19.0
	3	18.67	93.99	18.67
	4	18.67	93.99	18.67
4	1	18.69	93.91	na
	2	18.8	93.5	~19.0
	3	18.70	93.91	19.00
	4	18.70	93.91	19.00

na = not available

The HEU and natural-uranium hemispherical shells used in these experiments were part of an extensive set of shells fabricated for measurements of bare and reflected HEU spheres. Some of these shells were also used in the Godiva-shell experiment, a bare HEU assembly (see HEU-MET-FAST-001). More detailed geometry and mass data for the shells used in these experiments are presented in Table 3. The data in Table 3 are not needed for establishing the benchmark-model specifications, but are included for the sake of completeness.

Table 3. Geometry and Mass Data for Shells and Fillers.

Shell/Filler Number	HANSEN <sup>(a)</sup>			DRAWINGS <sup>(b)</sup>		
	ID (in.)	OD (in.)	Mass (kg)	ID (in.)	OD (in.)	Dwg. No.
HEU SHELLS						
HEU1	0.83	4.9456	19.426	0.83 <sup>+001</sup> <sub>-000</sub>	4.952 <sup>+000</sup> <sub>-002</sub>	19Y29215
HEU2	4.9567	5.4428	6.364	4.954 <sup>+004</sup> <sub>-000</sub>	5.444 <sup>+000</sup> <sub>-004</sub>	19Y29151
HEU3	4.9557	6.1043	17.065	4.954 <sup>+002</sup> <sub>-000</sub>	6.106 <sup>+000</sup> <sub>-002</sub>	19Y29215
NATURAL-URANIUM SHELLS						
N1	4.968	7.488	na	na	na	na
N2	5.455	5.478	na	na	na	na
N3	5.757	6.107	na	5.755±004	6.109±004	19Y29163
N4	6.117	7.493	na	6.114 <sup>+006</sup> <sub>-000</sub>	7.490 <sup>+006</sup> <sub>-000</sub>	19Y29156
N5	7.528	8.972	na	na	na	na
N6	8.999	10.000	na	8.980 <sup>+006</sup> <sub>-000</sub>	9.992 <sup>+000</sup> <sub>-006</sub>	19Y29156
N7	10.015	12.000	na	na	na	na
N8	na	na	na	4.960 <sup>+004</sup> <sub>-000</sub>	12.800±01	19Y29156
FILLERS						
F1	na	0.83	0.0704	(c)	0.828 <sup>+000</sup> <sub>-002</sub>	na
F2	0.34	0.83	0.084	na	na	19Y29215

na = not available

- (a) The data under the “HANSEN” column heading were obtained from notes written by Gordon Hansen (an experimenter) and from information presented in References 1 and 3. The diameters shown in these columns were adjusted about nominal values to give proper shell volumes that were calculated from measured masses and densities. The densities were 18.806 gm/cm<sup>3</sup> and 19.00 gm/cm<sup>3</sup> for HEU and natural uranium, respectively.
- (b) The data under the “DRAWINGS” column heading were obtained from engineering drawings referenced in the right-hand column.
- (c) cylindrical hole, 0.34 in. dia. x 0.26 in. high.

When describing the Godiva experiment Reference 3 states that the material density, 18.806±0.008 g/cm<sup>3</sup>, is the average measured by liquid immersion for some of the shells used in Godiva and a number of similarly fabricated parts.

Other characteristics of the Godiva material reported in Reference 3 include:

a <sup>234</sup>U content of 1.02 wt.% with no <sup>236</sup>U, and major trace impurities of ~160 ppm carbon, ~110 ppm silicon, and ~70 ppm iron.

#### **1.4 Supplemental Experimental Measurements**

No supplemental measurements were reported for these experiments.

## 2.0 EVALUATION OF EXPERIMENTAL DATA

Reference 1, published in 1952, appears to be the original report prepared for these specific experiments. Reference 2, published in 1958, is a compilation of many thinly reflected HEU spheres (24 configurations) and cylinders (32 configurations), including the experiments in this evaluation. In Reference 2 the material densities and enrichments were adjusted to a “standard” HEU concentration (93.5 wt.%  $^{235}\text{U}$ ) and density (18.8 gm/cm<sup>3</sup>) and the reported delayed-critical masses were normalized to these standard conditions.

Reference 3, published in 1969, presents reevaluated critical specifications for many Los Alamos fast-neutron systems, including three of the experiments in this evaluation. Case 2 in Tables 1 and 2 was not included in Reference 3 because “...the neutron multiplication attained is insufficient for reliable extrapolation to criticality.” The incentive for reexamination of these data was a subsequent generalization of reactivity coefficients which permitted correction for effects of filling gaps within the core and between the core and reflector. The specific correction procedure was not described. The reference states that “Although there are differences in detail, these critical specifications are equivalent to the published data to within the quoted probable errors.”

Finally, Reference 4, published in 1975, is a compilation of hundreds of Los Alamos experiments. Its introduction states that it was the purpose of the compilation to retrieve original data and give some means of judging the quality of the measurements. The abstract states that the compilation “modifies some of the old critical specifications that have been reevaluated.”

The critical specifications presented in Reference 4 were used to prepare the benchmark models used in this evaluation. It is the most recent compilation, includes previous reevaluations, and includes data for all 4 of the experiments being evaluated. From Tables 1 and 2 it can be seen that except for the critical mass, the basic data in Reference 4 are identical to those presented in Reference 3. The critical mass is presented in units of kg  $^{235}\text{U}$  in Reference 4, and kg U in the other references. Based on the given values of  $^{235}\text{U}$  wt.% the critical masses in References 3 and 4 are also identical to the 4 significant figures used.

Although the experiments were not brought to the delayed-critical state, the experimenters estimated the uncertainties in the extrapolation from the assembled masses to the reported critical masses. These uncertainties, along with other potential uncertainties in the basic material data and dimensions estimated in this evaluation, are used to derive corresponding uncertainties in the predicted values of  $k_{\text{eff}}$ . The resulting total  $k_{\text{eff}}$  uncertainties of approximately  $\pm 0.2\%$  to  $\pm 0.3\%$  are considered to be sufficiently small to judge these experiments acceptable for use as benchmarks.

### 2.1 Sensitivity Studies

Sensitivity studies were performed to determine the effect of various uncertainties in the reported experimental data on the value of  $k_{\text{eff}}$ . The calculations used the ONEDANT code with the SCALE4.3 27-energy-group ENDF/B-IV cross sections. The calculations were run with  $S_{16}$  quadrature,  $P_3$  scattering, and  $10^{-6}$  convergence criteria.

The calculated uncertainties in  $k_{\text{eff}}$  are summarized in Table 4.

Table 4. Calculated Uncertainties in  $k_{\text{eff}}$ .

Effect	Nominal Uncertainty Value	$k_{\text{eff}}$			
		Case Number			
		1	2	3	4
Uncertainty in Experimental $k_{\text{eff}}$					
Extrapolated Uranium Mass	See Section 2.1.1	$\pm 0.0013$	$\pm 0.0026$	$\pm 0.0013$	$\pm 0.0013$
Uncertainty in Benchmark-Model $k_{\text{eff}}$					
HEU Density	$\pm 0.12\%$	$\pm 0.0004$	$\pm 0.0004$	$\pm 0.0005$	$\pm 0.0006$
HEU Mass	$\pm 0.14\%$	$\pm 0.0004$	$\pm 0.0004$	$\pm 0.0004$	$\pm 0.0004$
$^{235}\text{U}$ Enrichment	$\pm 0.10\%$	$\pm 0.0005$	$\pm 0.0005$	$\pm 0.0005$	$\pm 0.0005$
Reflector Density	$\pm 0.12\%$	$\pm 0.0003$	$\pm 0.0003$	$\pm 0.0002$	$\pm 0.0001$
Reflector Thickness	See Section 2.1.5	$\pm 0.0001$	$\pm 0.0002$	$\pm 0.0002$	$\pm 0.0001$
Total Uncertainty	---	$\pm 0.0015$	$\pm 0.0027$	$\pm 0.0016$	$\pm 0.0016$

**2.1.1 Extrapolated Mass** - The reported extrapolated critical masses and their associated probable errors are shown in Table 1. The relative mass uncertainties inherent in the Table 1 data were rounded to 0.5% for cases 1, 3, and 4, and 1.0% for case 2. An uncertainty in the reported extrapolated critical mass causes an associated change in the core radius of the benchmark model based on the following relationship:

$$\text{core volume} = \text{extrapolated critical mass} / \text{HEU density}$$

The sensitivity calculations for extrapolated mass kept the HEU density constant, determined the change in core radius equivalent to the relative mass uncertainty from the relationship above, and calculated the associated change in  $k_{\text{eff}}$ . The resultant  $k_{\text{eff}}$  uncertainties are shown in Table 4. These are considered uncertainties in the experimental  $k_{\text{eff}}$ .

**2.1.2 Uranium Density** - The reported HEU density was derived from the measured material density of the shells and an adjustment to account for gaps inside the core and between the core and reflector. An uncertainty in the reported HEU density causes associated changes in the core radius and HEU atom densities of the benchmark model.

In the early 1950s a set of nesting hemispherical HEU-metal shells was fabricated for use in a series of bare and reflected HEU spherical assemblies including the bare Godiva-shell experiment. The

material used in the experiments being evaluated were either identical to, or the same type as that used in the Godiva-shell experiment.

Reference 3 states that the average material density, as measured by liquid immersion, for some of the Godiva “shells and a number of similarly fabricated parts” was  $18.806 \pm 0.008 \text{ g/cm}^3$  (equivalent to a relative error of  $\pm 0.04\%$ ).

To estimate the uncertainty in the gap adjustment, the relative difference between the core density reported in Reference 4 and the measured material density of  $18.806 \text{ g/cm}^3$  was calculated for each configuration. These differences ranged from 0.3% to 0.7% with an average value of 0.55%. Using the average relative adjustment of 0.55% and an assumed error of 20% (judged to be conservative), an adjustment uncertainty of 0.11% was derived.

The total HEU-density uncertainty used in the sensitivity study was estimated to be the square root of the sum of the squares of the two components which equals 0.12%. Associated changes in core radius and atom densities were determined and included in the calculations. The resulting changes in  $k_{\text{eff}}$  for a 0.12% change in the HEU density are shown in Table 4.

**2.1.3 Uranium Mass** - An error in the measured mass of the HEU shells manifests itself as an error in the extrapolated critical mass reported by the experimenters and, in turn, in the calculated core radius used in the benchmark model (see relationship presented in Section 2.1.1). Because the critical mass and HEU density are independent experimental parameters that effect the benchmark model, they must both be considered in the sensitivity studies.

Based on a discussion with a Los Alamos expert in statistics and nuclear materials accounting, a value of  $\pm 0.10\%$  was assumed for the relative random and systematic weighing errors.<sup>a</sup> The total weighing error was estimated to be the square root of sum of the squares of the random and systematic errors which equals  $\pm 0.14\%$ . The associated changes in the core radii were determined and included in the  $k_{\text{eff}}$  calculations. The corresponding calculated error in  $k_{\text{eff}}$  for a 0.14% change in the critical mass was 0.0004 for all configurations..

**2.1.4 Uranium Enrichment** - The reported values of  $^{235}\text{U}$  and  $^{234}\text{U}$  wt.% are presented in Section 1.3. A long-term Los Alamos chemist who is familiar with mass spectrometry measurements made in the 1950s stated that a value of 0.1% would be a reasonable estimate of the uncertainty in the HEU  $^{235}\text{U}$  weight fraction.<sup>b</sup>

The sensitivity study assumed a  $^{235}\text{U}$  enrichment uncertainty of 0.1%. The  $^{234}\text{U}$  wt.% was kept constant at 1.02, and the  $^{238}\text{U}$  value was adjusted to keep the total wt.% at 100. The resulting uncertainty in  $k_{\text{eff}}$  for a  $\pm 0.1\%$  change in the  $^{235}\text{U}$  enrichment was  $\pm 0.0005$  for all configurations.

**2.1.5 Reflector Uncertainty** - The specified independent parameters used to derive the dimensions and composition of the natural-uranium reflector in the benchmark model are the reflector density and thickness. The uncertainties associated with physically measuring these parameters are

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<sup>a</sup> Personal communication with Tom Burr, Group NIS-7, LANL, May, 1997.

<sup>b</sup> Personal communication with Darryl Jackson, LANL, July, 1997.

relatively small. As discussed in Section 2.1.2, the measurement of the of the HEU physical density had a reported uncertainty of 0.04%, which should be similar in magnitude for the natural-uranium components.

In Reference 3, G. E. Hansen describes how the volume and nominal dimensions of the Godiva shells were determined. Because masses and density are known more precisely than volumes, the volumes were calculated from the mass and density, and the radii (as shown in the Hansen column in Table 3) were determined by distributing the calculated volume around the nominal mean radius from the drawings. It can be assumed that the same method was used for the reflector shells as well.

The relative error in the calculated volume is then the square root of the sum of the squares of the relative density and weighing errors, which is 0.147%. The relative error of the shell thickness is, to first order, the same as the volume relative error, namely 0.147%.

Thus, the relative error in the sensitivity study for Cases 1 and 4 used a thickness relative error of 0.147%. For Cases 2 and 3, which had multi-shell reflectors, the thickness relative error was increased by 2 to 0.208% to account for the need to subtract information (outer and inner radius, respectively) from the outer and inner shells.

Additional uncertainties exist in the adjustments applied to account for gaps inside the reflector. These should be minimal in Cases 1 and 4 whose reflectors contain only a single shell, but may be more significant for Cases 2 and 3 which contain multi-shell reflectors. In the absence of more definitive information, the gap-adjustment uncertainty of 0.11% previously derived for the core adjustments (see Section 2.1.2) was applied to both the reflector density and size uncertainties for all configurations.

A summary of the assumed reflector uncertainties are presented in Table 5.

Table 5. Assumed Reflector Uncertainties.

Effect	Relative Uncertainty (%)			
	Case Number			
	1	2	3	4
Material Density				
Physical Density	0.040	0.040	0.040	0.040
Gap Adjustment	0.110	0.110	0.110	0.110
Total Uncertainty	0.120	0.120	0.120	0.120
Thickness				
Physical Thickness	0.147	0.208	0.208	0.147
Gap Adjustment	0.110	0.110	0.110	0.110
Total Uncertainty	0.184	0.235	0.235	0.184

The  $k_{\text{eff}}$  changes corresponding to the uncertainties of the reflector parameters are shown in Table 4. They were treated as independent variables.

**2.1.6 Total Uncertainty of Benchmark-Model  $k_{\text{eff}}$**  - An estimate of the total uncertainty of the benchmark-model  $k_{\text{eff}}$  was derived from the square root of the sum of the squares of the component uncertainties. The values of the total uncertainty are shown in Table 4.



### 3.0 BENCHMARK SPECIFICATIONS

#### 3.1 Description of Models

The benchmark model is based on the idealized critical data provided by the experimenters that included an extrapolation to the critical state and gap and central cavity corrections to represent a homogeneous HEU core surrounded by a close fitting natural-uranium reflector.

The effect on  $k_{\text{eff}}$  of the HEU impurities presented in Section 1.3 (with an equivalent reduction in the uranium density) was calculated to be less than 0.0001, and therefore judged to be negligible.

**Room Return** - External components that might potentially effect reactivity can be seen in Figure 1. These include the aluminum annular pedestal, the steel platen, hydraulic lift and support columns, and the steel beams comprising the platform and “A” frames. Further removed are the three neutron detectors, and finally the room concrete walls, floor, and ceiling. Accurately representing these components would require a detailed MCNP model. Standard one-million history runs would yield an uncertainty of approximately  $\pm 0.0008$   $k$ , greater than the expected value of the effect.

Data from the Godiva-shell experiment described in Reference 3 can be used to estimate the reactivity effect of the environment. The reported effect of incidental reflection including structure, walls, ceiling, and atmosphere for Godiva was  $+(0.29 \pm 0.15)\%$  of the critical mass. Using this percentage, along with the  $k_{\text{eff}}/\text{mass}$  ratio calculated for the current experiments ( $0.26\% \text{ } k/1.0\%$  mass), yields a  $k_{\text{eff}}$  change of  $0.0008 \pm 0.0004$ . Because the current configurations are reflected, the effect should be smaller. Because it would be extremely difficult to calculate an accurate bias for this effect, and the expected magnitude of the effect is within the predicted uncertainty of the benchmark-model  $k_{\text{eff}}$ , a bias has not been applied. However, an additional uncertainty of  $\pm 0.0005$  has been applied to the benchmark-model  $k_{\text{eff}}$  uncertainty to account for the uncertainty due to the effects of room return.

#### 3.2 Dimensions

The dimensions of the benchmark models are shown in Table 6. They were calculated from the critical masses, uranium densities, and  $^{235}\text{U}$  wt.%'s reported in Reference 4 and presented in Sections 1.2 and 1.3.

Table 6. Benchmark Model Dimensions.

Case Number	HEU Core Outer Radius (cm)	Reflector Outer Radius (cm)
1	6.32598	16.30818
2	6.38531	15.32611
3	6.97659	11.40127
4	7.75520	9.49002

### 3.3 Material Data

The atom densities calculated from the material information reported in Reference 4 and presented in Section 1.3 are shown in Table 7. The atom fractions for natural uranium were taken from the Chart of the Nuclides, Fourteenth Edition, General Electric Co., 1989.

Table 7. Isotopic Atom Densities for Core and Reflector.

Isotope	Atomic Density (atoms/barn-cm)			
	Case 1	Case 2	Case 3	Case 4
HEU Core				
$^{234}\text{U}$	$4.9053 \times 10^{-4}$	$4.9210 \times 10^{-4}$	$4.9001 \times 10^{-4}$	$4.9079 \times 10^{-4}$
$^{235}\text{U}$	$4.4965 \times 10^{-2}$	$4.5109 \times 10^{-2}$	$4.4960 \times 10^{-2}$	$4.4994 \times 10^{-2}$
$^{238}\text{U}$	$2.4019 \times 10^{-3}$	$2.4096 \times 10^{-3}$	$2.3568 \times 10^{-3}$	$2.3984 \times 10^{-3}$
Natural-Uranium Reflector				
$^{234}\text{U}$	$2.6438 \times 10^{-6}$	$2.6438 \times 10^{-6}$	$2.5979 \times 10^{-6}$	$2.6438 \times 10^{-6}$
$^{235}\text{U}$	$3.4610 \times 10^{-4}$	$3.4610 \times 10^{-4}$	$3.4009 \times 10^{-4}$	$3.4610 \times 10^{-4}$
$^{238}\text{U}$	$4.7721 \times 10^{-2}$	$4.7721 \times 10^{-2}$	$4.6892 \times 10^{-2}$	$4.7721 \times 10^{-2}$

### 3.4 Temperature Data

The experiments were performed at room temperature.

### 3.5 Experimental and Benchmark-Model $k_{\text{eff}}$

All experiments were extrapolated to the critical state so that the experimental  $k_{\text{eff}}$  is equal to 1.0000. The experimental  $k_{\text{eff}}$  uncertainties are discussed in Section 2.1 and shown in Table 8.

The benchmark-model  $k_{\text{eff}}$  values, including uncertainties, are shown in Table 8. The uncertainties in the benchmark-model  $k_{\text{eff}}$  were estimated in Sections 2 and 3.1. They include a value of  $\pm 0.0005$  for the uncertainty due to effects of room return.

Table 8. Values and Uncertainties of Experimental and Benchmark-Model  $k_{\text{eff}}$ 's.

Case Number	Experimental $k_{\text{eff}}$	Benchmark-Model $k_{\text{eff}}$
1	$1.0000 \pm 0.0013$	$1.0000 \pm 0.0016$
2	$1.0000 \pm 0.0026$	$1.0000 \pm 0.0027$
3	$1.0000 \pm 0.0013$	$1.0000 \pm 0.0017$
4	$1.0000 \pm 0.0013$	$1.0000 \pm 0.0017$

#### 4.0 RESULTS OF SAMPLE CALCULATIONS

Calculated results for the United States codes are presented in Table 9. More detailed discussions of the calculations, including input listings, are given in Appendix A.

Table 9. Sample Calculation Results (United States).

Code (Cross Section Set) Case Number	KENO (Hansen-Roach)	KENO (27-Group ENDF/B-IV)	MCNP (Continuous Energy ENDF/B-V)	MCNP (Continuous Energy ENDF/B-VI)	TWODANT (27-Group ENDF/B-IV)
1	1.0016 $\times$ 0.0006	1.0072 $\times$ 0.0008	1.0054 $\times$ 0.0007	1.0032 $\times$ 0.0006	1.0091
2	1.0003 $\times$ 0.0008	1.0101 $\times$ 0.0009	1.0057 $\times$ 0.0006	1.0028 $\times$ 0.0007	1.0097
3	0.9980 $\times$ 0.0007	1.0042 $\times$ 0.0009	0.9992 $\times$ 0.0006	0.9979 $\times$ 0.0006	1.0060
4	1.0011 $\times$ 0.0008	1.0067 $\times$ 0.0009	0.9991 $\times$ 0.0006	0.9978 $\times$ 0.0006	1.0071

## 5.0 REFERENCES

1. G. E. Hansen and D. P. Wood, "Precision Critical-Mass Determinations for Oralloid and Plutonium in Spherical Tuballoy Tampers," LA-1356DEL, February, 1952.
2. G. E. Hansen, H. C. Paxton, and D. P. Wood, "Critical Masses of Oralloid in Thin Reflectors," LA-2203, July 1958.
3. G. E. Hansen and H. C. Paxton, "Reevaluated Critical Specifications of Some Los Alamos Fast-Neutron Systems," LA-4208, September 1969.
4. H. C. Paxton, "Los Alamos Critical-Mass Data," LA-3067-MS, December 1975.

**APPENDIX A: TYPICAL INPUT LISTINGS****A.1. KENO Input Listings****Hansen-Roach Cross Sections**

The input listings for the KENO-V.a calculations using 16-group Hansen Roach cross sections is shown below. The calculations used a total of 220 generations with 5000 neutrons per generation with the first 20 generations excluded from the statistics. The results are therefore based on 1 million active neutron histories.

Because there were no moderator atoms in the assembly,  $\phi_p$  was calculated using the atom densities and scattering cross sections of the uranium isotopes. The bounding cross section sets were linearly apportioned to approximate the calculated  $\phi_p$ .

**27-Group ENDF/B-IV Cross Sections**

Input listings for the KENO calculation using the CSAS module and the 27-group ENDF/B-IV SCALE cross-section library is shown below. The calculation used a total of 220 generations, with 3500 neutrons per generation, and skipped 20 generations. These results are therefore based on 700,000 active neutron histories.

-----  
Volume II

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KENO-V.a Input Listing (16-Energy-Group Hansen-Roach Cross Sections) for Case 1 of Table 9.

```
tu1 nat u refl 3.930 in. thick
READ PARAMETERS
LIB=41 GEN=220 NPG=5000 NSK=20 NL8=3000 LNG=15000000 TME=300
END PARAMETERS
READ MIXT
MIX=1 92400 4.9053e-4 92501 4.4965e-2 92860 2.4019e-3
MIX=2 92400 2.6438e-6 92508 2.5922e-4 92509 8.6876e-5 92856 4.7721e-2
END MIXT
READ GEOMETRY
SPHERE 1 1 6.3259772
SPHERE 2 1 16.308177
END GEOMETRY
END DATA
END
```

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HEU-MET-FAST-032

KENO-V.a Input Listing (16-Energy-Group Hansen-Roach Cross Sections) for Case 2 of Table 9.

```
tu2 nat u refl 3.520 in. thick
READ PARAMETERS
LIB=41 GEN=220 NPG=5000 NSK=20 NL8=3000 LNG=15000000 TME=300
END PARAMETERS
READ MIXT
MIX=1 92400 4.9210e-4 92501 4.5109e-2 92860 2.4096e-3
MIX=2 92400 2.6438e-6 92508 2.5922e-4 92509 8.6876e-5 92856 4.7721e-2
END MIXT
READ GEOMETRY
SPHERE 1 1 6.3853090
SPHERE 2 1 15.326109
END GEOMETRY
END DATA
END
```



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KENO-V.a Input Listing (16-Energy-Group Hansen-Roach Cross Sections) for Case 3 of Table 9.

```
tu3 nat u refl 1.742 in. thick
READ PARAMETERS
LIB=41 GEN=220 NPG=5000 NSK=20 NL8=3000 LNG=15000000 TME=300
END PARAMETERS
READ MIXT
MIX=1 92400 4.9001e-4 92501 4.4960e-2 92860 2.3568e-3
MIX=2 92400 2.5979e-6 92508 2.5473e-4 92509 8.5365e-5 92856 4.6892e-2
END MIXT
READ GEOMETRY
SPHERE 1 1 6.97659
SPHERE 2 1 11.401272
END GEOMETRY
END DATA
END
```

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KENO-V.a Input Listing (16-Energy-Group Hansen-Roach Cross Sections) for Case 4 of Table 9.

```
tu4 nat u refl 0.683 in. thick
READ PARAMETERS
LIB=41 GEN=220 NPG=5000 NSK=20 NL8=3000 LNG=15000000 TME=300
END PARAMETERS
READ MIXT
MIX=1 92400 4.9079e-4 92501 4.4989e-2 92860 2.4032e-3
MIX=2 92400 2.6438e-6 92508 2.5922e-4 92509 8.6876e-5 92856 4.7721e-2
END MIXT
READ GEOMETRY
SPHERE 1 1 7.7551979
SPHERE 2 1 9.490018
END GEOMETRY
END DATA
END
```

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HEU-MET-FAST-032

KENO-V.a Input Listing (27-Energy-Group ENDF-B/IV Cross Sections) for Case 1 of Table 9.

```
=csas25
tu1 refl thick=3.93
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9053e-4 END
U-235 1 0.0 4.4965e-2 END
U-238 1 0.0 2.4019e-3 END
U-234 2 0.0 2.6438e-6 END
U-235 2 0.0 3.4610e-4 END
U-238 2 0.0 4.7721e-2 END
END COMP
READ PARAMETERS
GEN=220 NPG=3500 NSK=20 TME=300 tba=10
END PARAMETERS
READ GEOMETRY
SPHERE 1 1 6.32598
SPHERE 2 1 16.30818
END GEOMETRY
END DATA
END
```

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HEU-MET-FAST-032

KENO-V.a Input Listing (27-Energy-Group ENDF-B/IV Cross Sections) for Case 2 of Table 9.

```
=csas25
tu2 refl thick=3.52
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9210e-4 END
U-235 1 0.0 4.5109e-2 END
U-238 1 0.0 2.4096e-3 END
U-234 2 0.0 2.6438e-6 END
U-235 2 0.0 3.4610e-4 END
U-238 2 0.0 4.7721e-2 END
END COMP
READ PARAMETERS
GEN=220 NPG=3500 NSK=20 TME=300 tba=10
END PARAMETERS
READ GEOMETRY
SPHERE 1 1 6.38531
SPHERE 2 1 15.32611
END GEOMETRY
END DATA
END
```

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HEU-MET-FAST-032

KENO-V.a Input Listing (27-Energy-Group ENDF-B/IV Cross Sections) for Case 3 of Table 9.

```
=csas25
tu3 refl thick=1.742
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9001e-4 END
U-235 1 0.0 4.4960e-2 END
U-238 1 0.0 2.3568e-3 END
U-234 2 0.0 2.5979e-6 END
U-235 2 0.0 3.4009e-4 END
238 2 0.0 4.6892e-2 END
END COMP
READ PARAMETERS
GEN=220 NPG=3500 NSK=20 TME=300 tba=10
END PARAMETERS
READ GEOMETRY
SPHERE 1 1 6.97659
SPHERE 2 1 11.40127
END GEOMETRY
END DATA
END
```

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HEU-MET-FAST-032

KENO-V.a Input Listing (27-Energy-Group ENDF-B/IV Cross Sections) for Case 4 of Table 9.

```
=csas25
tu4 refl thick=0.693
27GROUPPDF4 INFHOMMEDIUM
U-234 1 0.0 4.9079e-4 END
U-235 1 0.0 4.4994e-2 END
U-238 1 0.0 2.3984e-3 END
U-234 2 0.0 2.6438e-6 END
U-235 2 0.0 3.4610e-4 END
U-238 2 0.0 4.7721e-2 END
END COMP
READ PARAMETERS
GEN=220 NPG=3500 NSK=20 TME=300 tba=10
END PARAMETERS
READ GEOMETRY
UNIT1
SPHERE 1 1 7.75520
SPHERE 2 1 9.49002
END GEOMETRY
END DATA
END
```

## **A.2 MCNP INPUT LISTINGS**

A typical input listing for the MCNP calculation using continuous-energy ENDF/B-V cross sections is shown below. The calculation employed a total of 110 generations and 10000 neutrons per generation. The first 10 generations were skipped so that the results are based on 1,000,000 active neutron histories.

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MCNP Input Listing for Case 1 of Table 9.

tu1 Natural Uranium 3.93 inch sphere reflector

1 1.04785743 -1  
2 2.0480697438 1 -2  
3 0 2

1 so 6.32598  
2 so 16.30818

imp:n 1 1 0

kcode 10000 1.0 10 110 4500 0

sdef erg=d1 rad=d2

sp1 -3.988 2.249

si2 0 6.32598

m1 92234.50c 4.9053e-4 92235.50c 4.4965e-2 92238.50c 2.4019e-3

m2 92234.50c 2.6438e-6 92235.50c 3.4610e-4 92238.50c 4.7721e-2

prdmp j 1000

print



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HEU-MET-FAST-032

MCNP Input Listing for Case 2 of Table 9.

tu2 Natural Uranium 3.52 inch sphere reflector

1 1.04801070 -1  
2 2.0480697438 1 -2  
3 0 2

1 so 6.38531  
2 so 15.32611

imp:n 1 1 0

kcode 10000 1.0 10 110 4500 0

sdef erg=d1 rad=d2

sp1 -3.988 2.249

si2 0 6.38531

m1 92234.50c 4.9210e-4 92235.50c 4.5109e-2 92238.50c 2.4096e-3

m2 92234.50c 2.6438e-6 92235.50c 3.4610e-4 92238.50c 4.7721e-2

prdmp j 1000

print

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HEU-MET-FAST-032

MCNP Input Listing for Case 3 of Table 9.

```
tu3  Natural Uranium 1.742 inch sphere reflector
1  1.04780681  -1
2  2.0472346888  1 -2
3  0 2

1  so 6.97659
2  so 11.40127
imp:n 1 1 0
kcode 10000 1.0 10 110 4500 0
sdef  erg=d1 rad=d2
sp1  -3.988 2.249
si2  0 6.97659
m1  92234.50c 4.9001e-4 92235.50c 4.4960e-2 92238.50c 2.3568e-3
m2  92234.50c 2.5979e-6 92235.50c 3.4009e-4 92238.50c 4.6892e-2
prtmp j 1000
print
```

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MCNP Input Listing for Case 4 of Table 9.

tu4 Natural Uranium 0.683 inch sphere reflector

1 1.04788319 -1  
2 2.0480697438 1 -2  
3 0 2

1 so 7.75520  
2 so 9.49002

imp:n 1 1 0

kcode 10000 1.0 10 110 4500 0

sdef erg=d1 rad=d2

sp1 -3.988 2.249

si2 0 7.75520

m1 92234.50c 4.9079e-4 92235.50c 4.4994e-2 92238.50c 2.3984e-3

m2 92234.50c 2.6438e-6 92235.50c 3.4610e-4 92238.50c 4.7721e-2

prdmp j 1000

print

### **A.3 ONEDANT Input Listing**

The input listings for the ONEDANT calculation using 27-group ENDF/B-IV SCALE cross sections is shown below. The ONEDANT code was run on the ORNL SCALE4.3 code system. The problem was run with  $S_{16}$  quadrature,  $P_3$  scattering, and a  $10^{-6}$  convergence criteria.

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CSAS Input File for Case 1 of Table 9.

```
tu1sig 3.93-in refl
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9053E-04 END
U-235 1 0.0 4.4965E-02 END
U-238 1 0.0 2.4019E-03 END
U-234 2 0.0 2.6438E-06 END
U-235 2 0.0 3.4610E-04 END
U-238 2 0.0 4.7721E-02 END
END COMP
MORE DATA
AXS=7
END DATA
END
```

ONEDANT Input Listing for Case 1 of Table 9.

```
tu1 3.93-in-reflector
/ ***** block i *****
igeom=sph ngroup=27 isn=16 niso= 2 mt=2 nzone= 2 im=2 it=160
maxscm=200000 maxlcm=900000 t
/ ***** block ii *****
xmesh= 0.0 6.325977 16.308177
xints=60 100
zones= 1 2 t
/
/ Block 3
lib=xstu1 writmxs=macbcd
chivec= .021 .188 .215 .125 .166 .180 .090 .014 .001 18z
maxord=3 ihm=42 iht=3 ihs=16 ititl=1 ifido=2 i2lp1=1 t
/
/ Block 4
matls=isos assign=matls t
/
/ Block 5
ievt=1 isct=3 ith=0 ibl=1 ibr=0 epsi=1.0e-6 norm=1 oitm=30 t
/
/ Block 6
zned=0 resdnt=1 massed=1 t
```

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HEU-MET-FAST-032

CSAS Input File for Case 2 of Table 9.

```
tu2sig 3.52-in refl
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9210E-04 END
U-235 1 0.0 4.5109E-02 END
U-238 1 0.0 2.4096E-03 END
U-234 2 0.0 2.6438E-06 END
U-235 2 0.0 3.4610E-04 END
U-238 2 0.0 4.7721E-02 END
END COMP
MORE DATA
AXS=7
END DATA
END
```

ONEDANT Input Listing for Case 2 of Table 9.

```
tu2 3.52-in-reflector
/ ***** block i *****
igeom=sph ngroup=27 isn=16 niso= 2 mt=2 nzone= 2 im=2 it=160
maxscm=200000 maxlcm=900000 t
/ ***** block ii *****
xmesh= 0.0 6.38531 15.32611
xints=60 100
zones= 1 2 t
/
/ Block 3
lib=xstu2 writmxs=macbcd
chivec= .021 .188 .215 .125 .166 .180 .090 .014 .001 18z
maxord=3 ihm=42 iht=3 ihs=16 ititl=1 ifido=2 i2lp1=1 t
/
/ Block 4
matls=isos assign=matls t
/
/ Block 5
ievt=1 isct=3 ith=0 ibl=1 ibr=0 epsi=1.0e-6 norm=1 oitm=30 t
/
/ Block 6
zned=0 resdnt=1 massed=1 t
```

CSAS Input File for Case 3 of Table 9.

```
tu3sig 1.742-in refl
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9001E-04 END
U-235 1 0.0 4.4960E-02 END
U-238 1 0.0 2.3568E-03 END
U-234 2 0.0 2.5979E-06 END
U-235 2 0.0 3.4009E-04 END
U-238 2 0.0 4.6892E-02 END
END COMP
MORE DATA
AXS=7
END DATA
END
```

ONEDANT Input Listing for Case 3 of Table 9.

```
tu3 1.742-in-reflector
/ ***** block i *****
igeom=sph ngroup=27 isn=16 niso= 2 mt=2 nzone= 2 im=2 it=120
maxscm=200000 maxlcm=900000 t
/ ***** block ii *****
xmesh= 0.0 6.97659 11.40127
xints=70 50
zones= 1 2 t
/
/ Block 3
lib=xstu3 writmxs=macbcd
chivec= .021 .188 .215 .125 .166 .180 .090 .014 .001 18z
maxord=3 ihm=42 iht=3 ihs=16 ititl=1 ifido=2 i2lp1=1 t
/
/ Block 4
matls=isos assign=matls t
/
/ Block 5
ievt=1 isct=3 ith=0 ibl=1 ibr=0 epsi=1.0e-6 norm=1 oitm=30 t
/
/ Block 6
zned=0 resdnt=1 massed=1 t
```

CSAS Input File for Case 3 of Table 9.

```
tu4sig 0.683-in refl
27GROUPNDF4 INFHOMMEDIUM
U-234 1 0.0 4.9079E-04 END
U-235 1 0.0 4.4994E-02 END
U-238 1 0.0 2.3984E-03 END
U-234 2 0.0 2.6438E-06 END
U-235 2 0.0 3.4610E-04 END
U-238 2 0.0 4.7721E-02 END
END COMP
MORE DATA
AXS=7
END DATA
END
```

ONEDANT Input Listing for Case 3 of Table 9.

```
tu4 0.683-in-reflector
/ ***** block i *****
igeom=sph ngroup=27 isn=16 niso= 2 mt=2 nzone= 2 im=2 it=110
maxscm=200000 maxlcm=900000 t
/ ***** block ii *****
xmesh= 0.0 7.75520 9.49002
xints=80 30
zones= 1 2 t
/
/ Block 3
lib=xstu4 writmxs=macbcd
chivec= .021 .188 .215 .125 .166 .180 .090 .014 .001 18z
maxord=3 ihm=42 iht=3 ihs=16 ititl=1 ifido=2 i2lp1=1 t
/
/ Block 4
matls=isos assign=matls t
/
/ Block 5
ievt=1 isct=3 ith=0 ibl=1 ibr=0 epsi=1.0e-6 norm=1 oitm=30 t
/
/ Block 6
zned=0 resdnt=1 massed=1 t
```